and (8) are written as

$$F_k(c_1, c_2, c_3) = \min_{w_{i,k+1}} \left\{ \sum_{i=0}^{3} \left(\frac{1}{2} \left[\left(\frac{c_{i+1} + c_i}{\Delta x} \right)^2 + \left(\frac{w_{i,k+1} - c_i}{\Delta y} \right)^2 \right] - c_i \right) \Delta x \, \Delta y + \right.$$

$$\left. + F_{k+1}(w_{1,k+1}, w_{2,k+1}, w_{3,k+1}) \right\} \qquad (k = 0, 1, \ldots 6)$$

$$F_7(c_1, c_2, c_3) = \sum_{i=0}^{3} \left(\frac{1}{2} \left[\left(\frac{c_{i+1} - c_i}{\Delta x} \right)^2 + \left(\frac{w_{i8} - c_i}{\Delta y} \right)^2 - c_i \right) \Delta x \, \Delta y$$

Results of the calculations are presented in Fig. 2. Membrane deflections at the section $x = 0.5$ are shown for $a = 0.04$ and $a = 0.07$ by curves *1* and *2*, respectively. The value of the functional at $a = 0.07$ turned out to be $J = -0.0151$. The results obtained are in good agreement with the results in [5], where the solution of an analogous problem was performed by the method of local variations.

In conclusion, let us note that dynamic programing can be applied to solve such a class of two-dimensional problems even for restrictions of more general type on the deformation.

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SPLITTING OF AN INFINITE ELASTIC WEDGE

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The problem of splitting an infinite elastic wedge with a thin perfectly rigid smooth plate is considered. The plate is driven in along the bisectrix of the wedge angle and a slot forms in front of it, when $a \leqslant r \leqslant b$. The wedge faces are either free or hinged.

Formulas defining the form of the slot surface and the normal stress intensity coefficient are obtained. Effective asymptotic methods developed in [1] as well as the mathematical apparatus of the Wiener-Hopf method [2] are employed in the course of solution.

1. Statement of the problem. Solution of the problem by approximating the function L. Let a thin perfectly rigid smooth plate of constant thickness $2h$ be driven into an elastic isotropic wedge bounded by the rays $\theta = \pm\alpha$ $(0 \leqslant r < \infty)$ along its bisectrix(see Fig.). In front of the plate, a slot is formed occupying the region $\{\theta = 0,\ a \leqslant r \leqslant b\}$. Wedge faces are either hinged (1) or stress-free (2). The boundary conditions of the problem have the form for $\theta = 0$

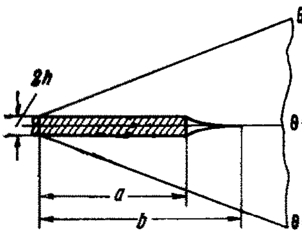

Fig. 1

$$u_\theta = \pm h\ (0 \leqslant r \leqslant a), \quad u_\theta = 0 \quad (b \leqslant r < \infty)$$

$$\sigma_\theta = 0\ (a < r < b), \qquad \tau_{r\theta} = 0 \quad (0 \leqslant r < \infty) \qquad (1.1)$$

for $\theta = \pm\alpha (0 \leqslant r < \infty)$

$$(1) \quad \tau_{r\theta} = u_\theta = 0, \qquad (2) \quad \tau_{r\theta} = \sigma_\theta = 0$$

and the stresses vanish at infinity.

The plus and minus signs correspond to the upper and lower boundary of the slot.

Our aim is to determine the form of the slot surface $v(r)$ and the normal stress intensity coefficient N the stresses appearing outside the slot on a line extended from it (for $\theta = 0$ and $r > b$). Obviously, by virtue of the symmetry we need only consider the region contained between the rays $\theta = 0$ and $\theta = \alpha$ $(0 \leqslant r < \infty)$.

Using a solution of the Lamé equations for the plane problem of the theory of elasticity in the form of Mellin integrals [1] and the boundary conditions (1.1), we can reduce the present problem to that of finding an unknown function $v(r) \equiv u_\theta(r,0)$ $(a \leqslant r \leqslant b)$ from the following integral equation:

$$\int_a^b v'(\rho)\, Q\left(\ln \frac{\rho}{r}\right) d\rho = 0 \qquad (a \leqslant r \leqslant b), \qquad Q(t) = \int_0^\infty L(u, \alpha) \sin(ut)\, du \qquad (1.2)$$

We have the following expressions for the function $L(u, \alpha)$ for conditions (1) and (2), respectively

$$(1) \quad L(u, \alpha) = \frac{\operatorname{sh} 2u\alpha + u \sin 2\alpha}{\operatorname{ch} 2u\alpha - \cos 2\alpha}, \qquad (2) \quad L(u, \alpha) = 2\frac{\operatorname{sh}^2 u\alpha - u^2 \sin^2 \alpha}{\operatorname{sh} 2u\alpha + u \sin 2\alpha}$$

We note the following properties of $L(u, \alpha)$:

$$L(u, \alpha) \to 1 + O(e^{-2u\alpha}) \quad \text{for} \quad u \to \infty; \quad L(u, \alpha) \to c^{-1}\pi u + O(u^3) \quad \text{for} \quad u \to 0$$

The following are the expressions for the constant c for conditions (1) and (2), respectively:

$$(1) \quad c = \pi \frac{1 - \cos 2\alpha}{2\alpha + \sin 2\alpha} \quad (\alpha \neq \pi), \qquad (2) \quad c = \pi \frac{2\alpha + \sin 2\alpha}{2\alpha^2 - 2\sin^2 \alpha} \qquad (1.3)$$

We shall use the expression $\operatorname{th}(c^{-1}\pi u)$ where c is given by (1.3), as an approximation to the function $L(u, \alpha)$. Similar approximation was used in [3], while [4] gives the relative accuracy of this approximation for various values of the angle α. Taking this approximation into account we find from (1.2),

$$Q\left(\ln \frac{\rho}{r}\right) = \frac{c\sqrt{r^c \rho^c}}{\rho^c - r^c}, \qquad \int_a^b \frac{\sqrt{\rho^c}\, v'(\rho)}{\rho^c - r^c} d\rho = 0 \qquad (a \leqslant r \leqslant b) \qquad (1.4)$$

Performing the change of variable in the second expression of (1.4) according to the formulas $\xi = \rho^c$ and $x = r^c$, we obtain a singular integral equation whose inversion formula is known [5]. Applying this formula and returning to the former variables, we obtain

$$v'(r) = \frac{P r^{0.5c-1}}{\pi\sqrt{(r^c - a^c)(b^c - r^c)}} \qquad (a \leqslant r \leqslant b) \qquad (1.5)$$

where P is a constant to be determined. From (1.5) we obtain

$$v(r) = h + \int_a^r v'(\rho)\,d\rho = h + \frac{2P}{\pi c \sqrt{b^c}} F\left(\arcsin \frac{\sqrt{r^c - a^c}}{k\sqrt{r^c}},\ k\right) \tag{1.6}$$

$$k = \sqrt{1 - \varepsilon^c}, \quad \varepsilon = a / b$$

where $F(\delta,\ k)$ is an elliptic integral of the first kind.

Constant P can be found from the obvious condition

$$v(b) = 0 \tag{1.7}$$

From (1.6) and (1.7) we obtain after simple manipulations

$$P = -0.5\pi hc \sqrt{\varepsilon^c} K^{-1}(k) \tag{1.8}$$

where $K(k)$ denotes a complete elliptic integral of the first kind. The normal stress intensity coefficient N, the stresses appearing outside the slot on the line extended from it, is given by

$$N = \lim_{r \to b+0} \sqrt{r - b}\,\sigma_\theta(r,\ 0) = -\lim_{r \to b-0} \sqrt{b - r}\,\frac{E}{2(1 - \nu^2)}\,v' \tag{1.9}$$

where E denotes the Young's modulus and ν is the Poisson's ratio.

Inserting into the second relation of (1.9) $v'(r)$ in the form given by (1.5) and taking (1.8) into account, we obtain

$$N = \frac{Eh\sqrt{c}}{4(1 - \nu^2)\,kK(k)\sqrt{b}} \tag{1.10}$$

It can be shown that the solution (1.6),(1.10) tends to the exact solution of the problem as $\varepsilon \to 0$ and $\varepsilon \to 1$, for all $0 < \alpha < \pi$. Let e.g. the length of the slot $l = b - a$ be fixed and $\varepsilon \to 1$. The relative length of the plate will then tend to infinity and the effect of the wedge face on N can, therefore, be neglected. The corresponding problem will be that of splitting a plane with a semi-infinite plate, when a slot of length l forms in front of the plate. In this case we obtain from (1.10)

$$\lim_{\varepsilon \to 1} N = \frac{Eh\sqrt{c}}{2(1 - \nu^2)\,\pi\sqrt{l}} \lim_{\varepsilon \to 1} \frac{\sqrt{1 - \varepsilon}}{\sqrt{1 - \varepsilon^c}} = \frac{Eh}{2(1 - \nu^2)\,\pi\sqrt{l}} \tag{1.11}$$

The value of N obtained above agrees with that obtained in [6] in the course of investigation of splitting a plane with a semi-infinite plate.

Numerical computations show that the obtained relations (1.6) and (1.10) defining the function $v(r)$ and the magnitude of N, respectively, can safely be used for any values of ε, when $65° \leqslant \alpha \leqslant 155°$ for condition (1) at the wedge edges and when $85° \leqslant \alpha \leqslant 180°$ for condition (2). The solution will become exact in two cases: when condition (1) holds and $\alpha = {}^1/_2\,\pi$, and when condition (2) holds and $\alpha = \pi$. In the first case $c = 2$, in the second case $c = 1$. The first case corresponds to the problem of splitting a semiplane whose boundary is hinged to a plate which is imbedded in it to the distance a. Boundary conditions (1.1) indicate, that it also corresponds to the problem of splitting a plane with a plate of length $2a$, when slots of length l are formed on both sides of this plate. The solution obtained for this case agrees with that given in [7]. The second case corresponds to the problem of splitting a plane with a plate of length a. A slot of length l forms here on one side of the plate, while a rectilinear semi-infinite cut is formed on the other side.

2. Solution of the problem for small ε. Using the method developed in [1] we shall seek an asymptotic solution of (1.2) valid for small values of ε, in the form

$$v'(r) = v_1(r) v_2(r)\, v_0^{-1}(r) \qquad (a \leqslant r \leqslant b) \tag{2.1}$$

We find the functions $v_1(r)$ and $v_2(r)$ from the following integral equations:

$$\int_0^b v_1(\rho)\, Q\left(\ln\frac{\rho}{r}\right) d\rho = 0, \qquad \int_a^\infty v_2(\rho)\, Q\left(\ln\frac{\rho}{r}\right) d\rho = 0 \tag{2.2}$$

$$(0 \leqslant r \leqslant b) \qquad\qquad (a \leqslant r < \infty)$$

The function $v_0(r)$ represents the null term of the asymptotics $v_2(r)$ when $r/a \to \infty$. Simple transformation of variables reduces Eqs. (2.2) to a single Wiener-Hopf equation

$$\int_0^\infty \psi(\tau)\, Q(\tau - t)\, d\tau = 0 \qquad (0 \leqslant t < \infty) \tag{2.3}$$

and we also have

$$v_1(\rho) = \frac{b}{\rho}\psi\left(\ln\frac{b}{\rho}\right), \qquad v_2(\rho) = \frac{a}{\rho}\psi\left(\ln\frac{\rho}{a}\right) \tag{2.4}$$

Let us extend Eq. (2.3) to the whole interval $-\infty < t < \infty$, by introducing a new unknown function [2]

$$w_-(t) = \frac{i}{\pi}\int_0^\infty \psi(\tau)\, Q(\tau - t)\, d\tau \qquad (-\infty < t < 0) \tag{2.5}$$

Applying the Fourier transform to (2.3) and (2.5) we obtain

$$\Psi_+(s) L(s, \alpha) = W_-(s) \tag{2.6}$$

where $\Psi_+(s)$ and $W_-(s)$ are Fourier transforms of the functions $\psi(t)$ and $w_-(t)$, respectively.

To obtain a usable solution, we shall employ the following expression to approximate the function L

$$L(s, \alpha) = \frac{s\sqrt{s^2 + D^2}}{s^2 + R^2}\prod_{n=1}^{m}\frac{s^2 + A_n^2}{s^2 + B_n^2} \qquad \left(\frac{D}{R^2}\prod_{n=1}^{m}\frac{A_n^2}{B_n^2} = \frac{\pi}{c}\right) \tag{2.7}$$

Here D, A_n and B_n are real positive numbers chosen from the condition of the best approximation. Taking into account (2.7), we can write (2.6) in the form

$$\frac{s\Psi_+(s)\sqrt{s + iD}}{s + iR}\prod_{n=1}^{m}\frac{s + iA_n}{s + iB_n} = \frac{W_-(s)(s - iR)}{\sqrt{s - iD}}\prod_{n=1}^{m}\frac{s - iB_n}{s - iA_n} \tag{2.8}$$

Both parts of (2.8) coincide with some function $G(s)$ regular on the whole complex s-plane, in the general band of regularity $\Pi_1(0 < \operatorname{Im} s < \operatorname{Inf}(R, D, A_n, B_n)$.

Since $\psi(t) \sim t^{-1/2}$ for $t \to +0$, $\Psi_+(s) \sim s^{-1/2}$ for $s \to \infty$ in the upper semiplane. Consequently the left side of (2.8) assumes a constant value as $s \to \infty$. Similarly, we can show that the right side of (2.8) also becomes constant in the lower semiplane as $s \to \infty$. Therefore, by the Liouville's theorem $G(s) = A_* = \text{const}$ and we have

$$\Psi_+(s) = A_* \frac{s + iR}{\sqrt{s + iD}\, s}\prod_{n=1}^{m}\frac{s + iB_n}{s + iA_n} \tag{2.9}$$

In the following we shall only consider the case $B_n = A_n (n = 1, \ldots m)$. In this case from (2.9) we obtain

$$\psi(t) = -A\left(\frac{e^{-Dt}}{\sqrt{\pi t}} + \frac{\sqrt{c}}{\sqrt{\pi}}\operatorname{erf}\sqrt{Dt}\right) \qquad \left(A = -\frac{\sqrt{2\pi}}{\sqrt{i}}A_*\right) \tag{2.10}$$

Having found the functions $v_1(r)$, $v_2(r)$ and $v_0(r)$ from (2.10) and (2.4) and using the relation (2.1), we obtain

$$v'(r) = -A\frac{b}{r}\left[\left(\frac{r}{b}\right)^D\left(\pi\ln\frac{b}{r}\right)^{-1/2} + \frac{\sqrt{c}}{\sqrt{\pi}}\operatorname{erf}\left(D\ln\frac{b}{r}\right)^{1/2}\right] \times$$

$$\times\left[\frac{\sqrt{\pi}}{\sqrt{c}}\left(\frac{a}{r}\right)^D\left(\pi\ln\frac{r}{a}\right)^{-1/2} + \operatorname{erf}\left(D\ln\frac{r}{a}\right)^{1/2}\right] \qquad (a \leqslant r \leqslant b) \tag{2.11}$$

Inserting now $v'(r)$ into the left part of (1.6), we find

$$v(r)=h-Ab\left[\frac{2e^{-\omega}}{\sqrt{\pi c}}\operatorname{arc\,tg}\left(\frac{\ln(r/a)}{\ln(b/r)}\right)^{1/2}+\right.\qquad(\omega=-D\ln\varepsilon)$$

$$\left.+\frac{1}{\sqrt{D}}\operatorname{erf}\left(D\ln\frac{b}{r}\right)^{1/2}\operatorname{erf}\left(D\ln\frac{r}{a}\right)^{1/2}+\frac{2}{\sqrt{\pi}}J_1(r)+\frac{\sqrt{c}}{\sqrt{\pi}}J_2(r)\right]\tag{2.12}$$

$$J_1(r)=\int_{\ln(b/r)}^{\omega/D}\frac{\operatorname{erf}\sqrt{\omega-Dt}}{\sqrt{t}\exp(Dt)}dt,\qquad J_2(r)=\int_{\ln(b/r)}^{\omega/D}\operatorname{erf}\sqrt{Dt}\operatorname{erf}\sqrt{\omega-Dt}\,dt$$

The constant A is obtained (after the necessary computations) from the condition (1.7)

$$A=\frac{h}{b}\left[\frac{\sqrt{\pi}e^{-\omega}}{\sqrt{c}}+\frac{2}{\sqrt{\pi}}J_1(b)+\frac{\sqrt{c}}{\sqrt{\pi}}J_2(b)\right]^{-1}\tag{2.13}$$

Approximate values of the integrals $J_i(b)$ $(i=1,2)$ whose relative error does not exceed 0.1% for $\omega=2.5$, are obtained from the following expressions:

$$J_1(b)=(B-\operatorname{erf}(^1/_2\sqrt{2\omega}))\frac{\sqrt{\pi}}{\sqrt{D}}\operatorname{erf}(^1/_2\sqrt{2\omega})-\frac{\sqrt{2}e^{-\omega}}{\sqrt{\pi D}}\left(\frac{13}{6}-\frac{1}{\omega}\right)$$

$$J_2(b)=\frac{1}{2D}\left\{B\left[(\omega-1)\operatorname{erf}(^1/_2\sqrt{2\omega})+\frac{\sqrt{2\omega e^{-\omega}}}{\sqrt{\pi}}\right]+\frac{\sqrt{2}e^{-\omega}}{\pi}\left(\frac{13}{3}-\frac{4}{\omega}\right)-\right.$$

$$\left.-\frac{\sqrt{e^{-\omega}}}{\sqrt{\pi\omega^{i}}}\left(5-\frac{4}{\omega}\right)\operatorname{erf}(^1/_2\sqrt{2\omega})\right\},\qquad B=2\operatorname{erf}\sqrt{\omega}+\left(2-\frac{1}{\omega}\right)\frac{e^{-\omega}}{\sqrt{\pi\omega}}$$

(The above expressions tend to exact values of $J_i(b)$ with increasing ω).

It can easily be confirmed that

$$J_1(\sqrt{ab})=\frac{1}{2}J_1(b)-\frac{\sqrt{\pi}}{2\sqrt{D}}[\operatorname{erf}(^1/_2\sqrt{2\omega})]^2,\qquad J_2(\sqrt{ab})=\frac{1}{2}J_2(b)\tag{2.14}$$

Taking into account (2.13) and (2.14), we obtain from (2.12)

$$v(\sqrt{ab})={}^1/_2h$$

Relation (2.11) and the second relation of (1.9) yield the normal stress intensity coefficient N

$$N=\frac{AE\sqrt{b}}{2\sqrt{\pi}(1-\nu^2)}\left(\operatorname{erf}\sqrt{\omega}+\frac{\sqrt{D}e^{-\omega}}{\sqrt{\omega c}}\right)\tag{2.15}$$

The accuracy of the expressions (2.12) and (2.15) just obtained, increases with decreasing ε. Let us determine the value of N for $\varepsilon\ll1$. Inserting the value of the constant A into (2.15), we find

$$\lim_{\varepsilon\to0}N\ln\varepsilon=-\frac{Eh}{2\sqrt{bc}(1-\nu^2)}$$

Consequently, when $\varepsilon\ll1$, we have

$$N=-\frac{Eh}{2\sqrt{bc}(1-\nu^2)\ln\varepsilon}$$

The above value for N can also be obtained from (1.10).

3. Solution of the problem for large λ. Performing a change of variables

$$r=a\exp\frac{1+x}{\lambda},\qquad\rho=a\exp\frac{1+\xi}{\lambda}\qquad\left(\lambda=-\frac{2}{\ln\varepsilon}=\frac{2D}{\omega}\right)$$

in (1.2), we obtain the following integral equation with a difference kernel dependent on the parameter λ

$$\int_{-1}^{1} \varphi(\xi)\, Q\left(\frac{\xi - x}{\lambda}\right) d\xi = 0 \qquad (\varphi(\xi) = \rho v'(\rho)) \tag{3.1}$$

When λ are large, the kernel of (3. 1) can be represented by

$$Q(t) = \frac{1}{t} + \sum_{i=0}^{\infty} \beta_i t^{2i+1} \tag{3.2}$$

Constants β_i are given by

$$\beta_i = \frac{(-1)^i}{(2i+1)!} \int_0^{\infty} [L(u, \alpha) - 1]\, u^{2i+1}\, du \qquad (i = 0, 1, \ldots)$$

Values of β_i are given in [8] for certain particular cases. Taking into account (3. 2), we can transform Eq. (3. 1) into

$$\int_{-1}^{1} \frac{\varphi(\xi)\, d\xi}{x - \xi} = -\sum_{i=0}^{\infty} \frac{\beta_i}{\lambda^{2i+2}} \int_{-1}^{1} \varphi(\xi)(x - \xi)^{2i+1}\, d\xi \tag{3.3}$$

We shall seek the solution of (3. 3) in the form of a series in powers of λ^{-2}. We omit the intermediate operations similar to those in [1 and 8], and give the final expression defining the function $\varphi(x)$

$$\varphi(x) = \pi^{-1}(1 - x^2)^{-1/2} T \Phi(x) \tag{3.4}$$

$$\Phi(x) = 1 + (x^2 - {}^1\!/_2)\beta_0\lambda^{-2} + (x^4 + x^2 - {}^7\!/_8)\beta_1\lambda^{-4} + [({}^9\!/_8 x^2 - {}^3\!/_{16})\beta_0\beta_1 + (x^6 + $$
$$+ {}^9\!/_2 x^4 - {}^3\!/_4 x^2 - {}^{13}\!/_8\, \beta_2]\lambda^{-6} + O(\lambda^{-8})$$

Returning now to the former notation and variables, we find

$$v'(r) = \frac{T}{\pi\lambda r}\left(\ln\frac{r}{a}\ln\frac{b}{r}\right)^{-1/2} \Phi(\chi) \qquad \left(\chi = \lambda \ln\frac{r}{\sqrt{ab}}\right) \tag{3.5}$$

The left part of (1. 6) and the condition of boundedness of the function $v(r)$ (1. 7) yield $v(r)$ and the constant T

$$v(r) = \frac{h}{\pi}\left\{\arccos\chi + \left[\frac{\beta_0}{2\lambda^2} + \left(\frac{7}{8} + \frac{1}{4}\chi^2\right)\frac{\beta_1}{\lambda^4} + \frac{3}{16\lambda^6}\beta_0\beta_1 + \right.\right.$$
$$\left.\left. + \left(\frac{13}{8} + \frac{4}{3}\chi^2 + \frac{1}{6}\chi^4\right)\frac{\beta_2}{\lambda^6}\right]\chi\sqrt{1 - \chi^2} + O(\lambda^{-8})\right\}, \qquad T = -\lambda h \tag{3.6}$$

Inserting now $v'(r)$ in the form given by (3. 5) into the second relation of (1. 9) and taking into account the value of T just obtained, we find

$$N = \frac{Eh\sqrt{\lambda}\,\Phi(1)}{2\pi\sqrt{2b}\,(1 - \nu^2)} \tag{3.7}$$

which, on taking λ to infinity, gives a value for N agreeing with that obtained in Sect. 1 in the form of (1. 11).

It can be shown that the series (3. 2) converges absolutely when $t < 2\alpha$. Consequently, results obtained in this section are valid for $1 / \alpha < \lambda < \infty$. In practice, the relations (3. 6) and (3. 7) become usable when $2 / \alpha < \lambda < \infty$.

Numerical results obtained in Sects. 2 and 3 for the determination of $v(r)$ and N cover the whole range of variation of the parameter $0 \leqslant \lambda < \infty$, thus representing a complete solution of the problem. The simpler solution (1. 6), (1. 10) can be used for all λ when either $65° \leqslant \alpha \leqslant 155°$ and conditions (1), or $85° \leqslant \alpha \leqslant 180°$ and conditions (2) hold at the wedge faces.

Values of the quantities $N_* = (1 - \nu^2)\sqrt{b}(Eh)^{-1} N$ and $v_* = h^{-1}v(\sqrt{ab})$ computed according to the formulas given in Sects. 1, 2 and 3 for $\lambda = 2$, $\alpha = {}^1\!/_2\pi$ and conditions (2) at the wedge faces are, respectively, $N_* = 0.151$, 0.152, 0.151 and $v_* = 0.500$, 0.500, 0.500.

Values of the constants used in the present case, are:

$\beta_0 = -{}^5/_{12}, \quad \beta_1 = {}^{11}/_{180}, \quad \beta_2 = -{}^{239}/_{12096}, \; D = 2.549, \quad A_n = B_n \; (n = 1, \ldots, m).$

The error of the approximation (2.7) does not exceed 3% for all $0 \leqslant \operatorname{Re} s < \infty$.

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A GENERALIZATION OF FOURIER'S INTEGRAL THEOREM AND ITS APPLICATIONS

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1. Solution of certain classes of the boundary value problems of mathematical physics for a two-layer medium demands that the given function be expanded into an integral in terms of the functions

$$\varphi(x, \lambda) = \begin{cases} \mu \sin \sqrt{\beta_1}\, \lambda x & (0 < x < l) \\ \sin \sqrt{\beta_1} \lambda l \cos \sqrt{\beta_2} \lambda (x - l) + \delta \cos \sqrt{\beta_1}\, \lambda l \sin \sqrt{\beta_2} \lambda (x - l) & (l < x < \infty) \end{cases} \tag{1.1}$$

which are eigenfunctions of the following singular boundary value problem:

$$\varphi'' + \beta_1 \lambda^2 \varphi = 0 \quad (0 < x < l), \quad \varphi'' + \beta_2 \lambda^2 \varphi = 0 \quad (l < x < \infty) \tag{1.2}$$

$$\varphi(0) = 0, \quad \varphi(\infty) < \infty, \quad \varphi(l - 0) = \mu\varphi(l + 0), \quad \varphi'(l - 0) = \nu\varphi'(l + 0)$$

The fundamental result of the present investigation can be stated in the form of the following theorem: if $f(x)$ is a piece-wise continuous function absolutely integrable on the interval $(0, \infty)$ and possessing a bounded variation in this interval, then

$$\frac{2}{\pi} \int_0^\infty \frac{\varphi(x, \lambda)}{\omega(\lambda)} d\lambda \int_0^\infty f(\xi) r(\xi) \varphi(\xi, \lambda) d\xi = \begin{cases} {}^1/_2 [f(x - 0) + f(x + 0)] & (x \neq l) \\ [\delta f(l - 0) + \mu f(l + 0)] / 1 + \delta & (x = l) \end{cases} \tag{1.3}$$